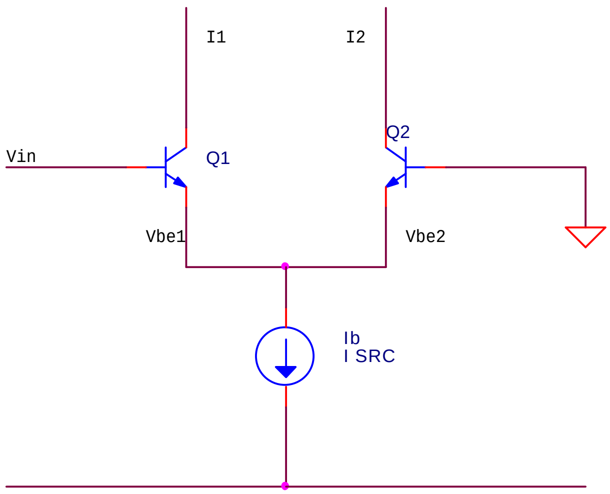
Secrets of OTAs

The Operational Transconductance Amplifier is one of the more useful components in making electronic music circuits. It is, in fact, probably the most used special purpose component. You could find one of these in almost every synthesizer module. An OTA is basically a two quadrant multiplier. It multiplies an input voltage by a control current and produces a current as its output.

The OTA's transfer function is that of a differential transistor pair, which is pretty convenient, although, a bit tough to analyze.



The above figure shows a basic differential pair. So, we can start by writing the following equations:

$\mathbf{I_B = I_1 + I_2}$ (1)

$\mathbf{I_1 = I_S * e^{(VBE1/VT)}}$ (2) $\mathbf{I_S}$ is the collector saturation current, **VT** is the thermal voltage (0.026V at room temperature).

$\mathbf{I_2 = I_S * e^{(VBE2/VT)}}$ (3)

We can now substitute equations (2) and (3) into equation (1) and solve for $\mathbf{I_S}$.

$\mathbf{I_B = I_S * e^{(VBE1/VT)} + I_S * e^{(VBE2/VT)}}$

$\mathbf{I_S = I_B /(e^{(VBE1/VT)} + e^{(VBE2/VT)})}$ (4)

Now we substitute equation (4) back into equations (2) and (3) and we get:

$\mathbf{I_1 = I_B * e^{(VBE1/VT)} /(e^{(VBE1/VT)} + e^{(VBE2/VT)})}$ (5)
$\mathbf{I_2 = I_B * e^{(VBE2/VT)} /(e^{(VBE1/VT)} + e^{(VBE2/VT)})}$ (6)

Equations (5) and (6) can be further simplified by using the following identity:

$$\mathbf{e^{a}/(e^{a} + e^{b}) = 1/(1 + e^{(b-a)})}$$

So equations (5) and (6) will end up looking like this:

$$\mathbf{I_1 = I_B/(1 + e^{(VBE2-VBE1)/VT})} \qquad (7)$$
$$\mathbf{I_2 = I_B/(1 + e^{(VBE1-VBE2)/VT})} \qquad (8)$$

And now, for some simplifying assumptions. It should be noticed that $V_{BE1} – V_{BE2}$ is equal to $V_{IN} – V_2$. V_2 is the voltage at the base of Q2. It can also be seen in that diagram that the base of Q2 is at ground level, so $V_2 = 0$. This means we can write the following equations:

$$\mathbf{V_{BE1} – V_{BE2} = V_{IN}} \qquad (9)$$
$$\mathbf{V_{BE2} – V_{BE1} = -V_{IN}} \qquad (10)$$

We can now use (9) and (10) to simplify (7) and (8) even further.

$$\mathbf{I_1 = I_B/(1 + e^{-Vin/VT})} \qquad (11)$$
$$\mathbf{I_2 = I_B/(1 + e^{Vin/VT})} \qquad (12)$$

Now we come to the part where you will find out how good your math skills are (well, for some of us, this isn’t easy). The output will be the difference between I1 and I2.

$$\mathbf{I_O = I_1 – I_2 = I_B/(1 + e^{-Vin/VT}) - I_B/(1 + e^{Vin/VT})} \qquad (13)$$

Now simplifying this equation is going to take just a little imagination. There may be more elegant ways of doing this, but this is how I approached the problem. If you look at the above equations (11) and (12) you will note a property about the. If $V_{IN} = 0$, then both (11) and (12) reduce to $I_B/2$. So what you may say. What this means is the following.

$$\mathbf{1/(1+e^{-X}) – 0.5 = 0.5 – 1/(1+e^{X})} \qquad (14)$$

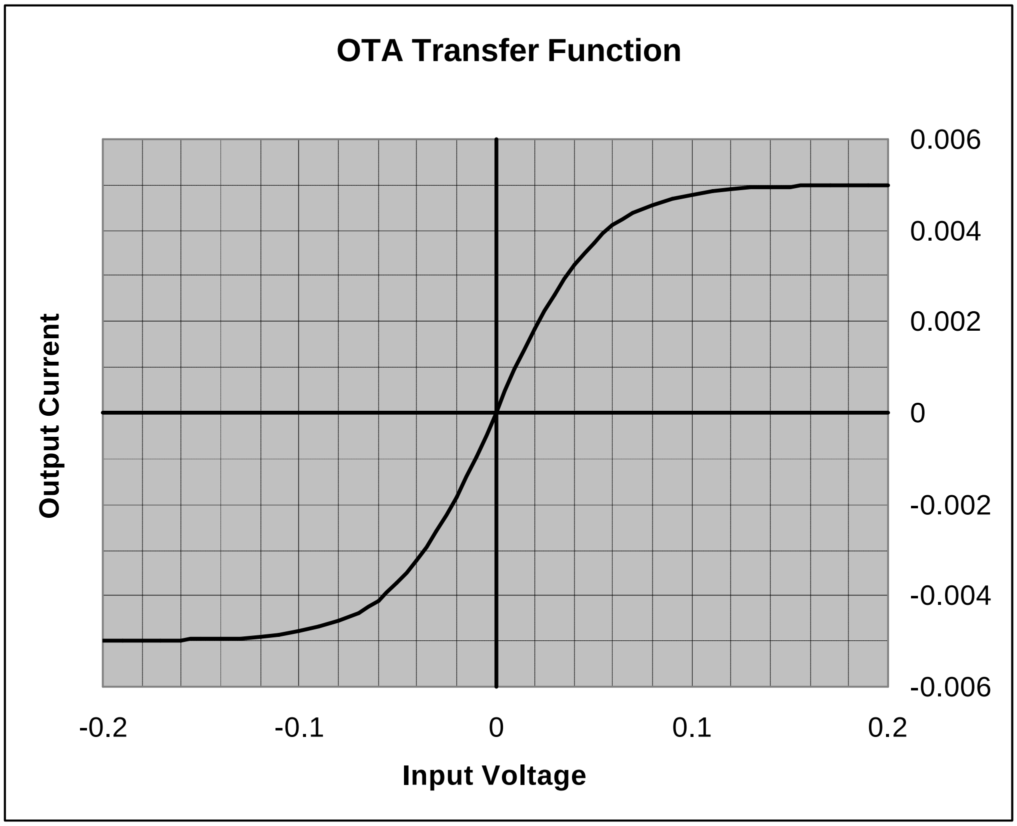
We can use the identity in (14) to help us further simplify (13) and make it more readable. By using (14) we can then write the following equation:

$$\mathbf{1/(1 + e^{-Vin/VT}) = 1 – 1/(1+e^{Vin/VT})} \qquad (15)$$

So, we can factor out the IB term in (13) and then substitute in (15) and we will get:

$$\mathbf{I_O = I_B – I_B/(1+e^{Vin/VT}) - I_B/(1+e^{Vin/VT})}$$

$$\mathbf{I_O = I_B – 2I_B/(1+e^{Vin/VT})} \qquad (16)$$



So, this is the final form of the transfer function for a differential pair. Now, to get the gain, which is the forward transconductance, we need to differentiate I_O with respect to V_{IN}. So, I am sorry I am going to have to do this to you, but, this involves calculus. For those who are not familiar with calculus, taking the differential of a function is just a way to write an equation that expresses the slope. This is, of course, a simplistic way of stating this, but I think you will understand the results even if you don't understand how I get there. Looking at equation (16), you will note that the first term is IB, which is a constant, so its slope is 0, in case you are wondering what happened to it.

Now the part of equation (16) that we are interested in is a quotient, so we will need to use the rule for differentiating a quotient.

$$d/dx\ (u/v) = (v\ du/dx - u\ dv/dx)/v^2 \qquad (17)$$

You will find the above formula in any calculus text or table of integrals. It may not be readily recognizable in the format I put it in above.

So, looking again at equation (16) we can make the following assignments:

$$u = 2 * I_B \qquad (18)$$
$$v = 1 + e^{Vin/VT} \qquad (19)$$
$$v^2 = 1 + 2e^{Vin/VT} + e^{2Vin/VT} \qquad (20)$$

$$\mathbf{du/dVin = 0} \qquad (21)$$
$$\mathbf{dv/dVin = e^{Vin/VT}/VT} \qquad (22)$$

So we can now differentiate equation (16) by substituting (18) -> (22) into equation (17).

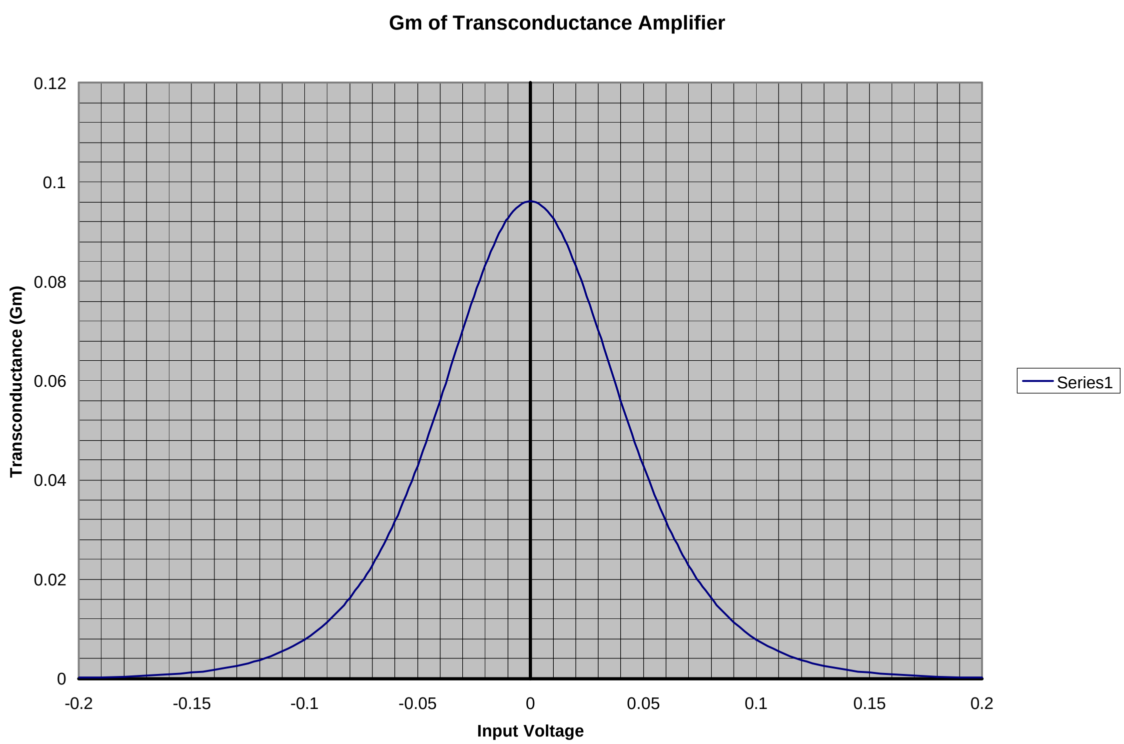
$$\mathbf{dI_O/dV_{IN} = -2 * IB * e^{Vin/VT} / VT * (1 + 2e^{Vin/VT} + e^{2Vin/VT}) = G_M} \qquad (23)$$

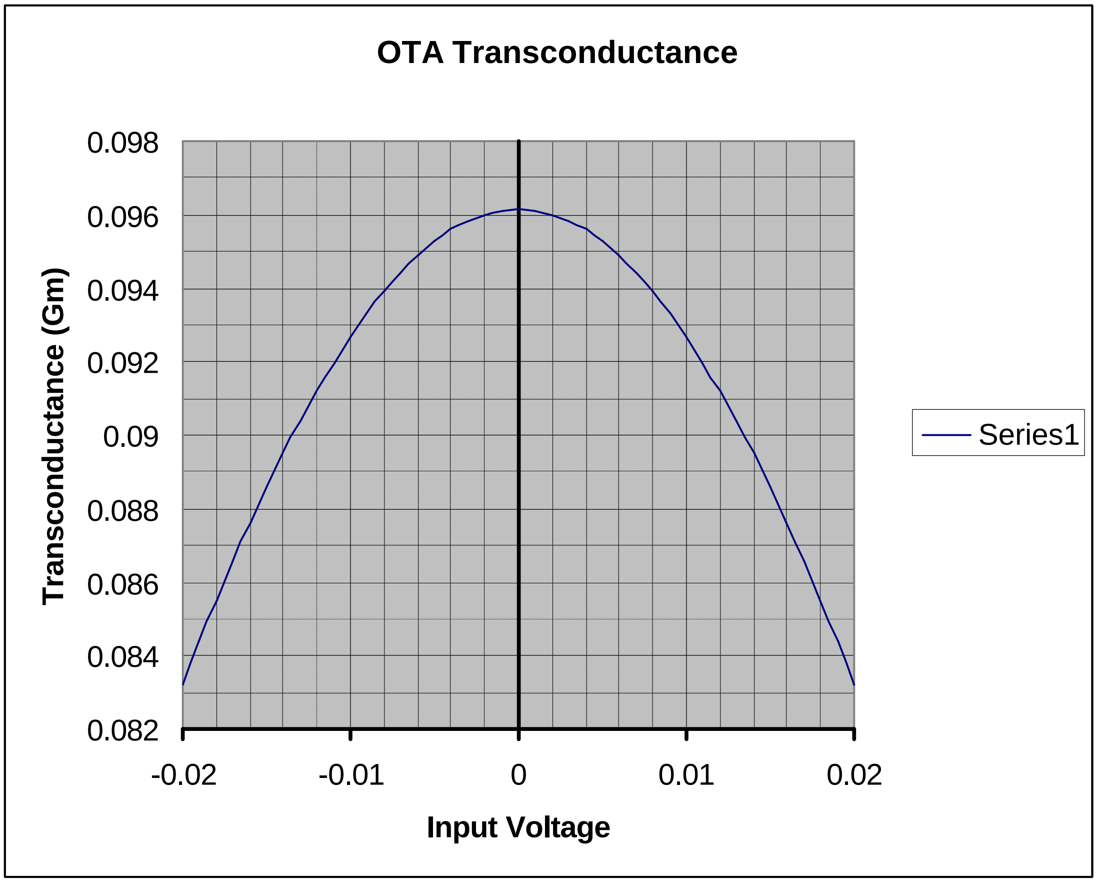
We can evaluate equation (23) at V_{IN}=0v to get the nominal value of GM. When we do so we get:

$$\mathbf{G_M = I_B / (2 * V_T)} \qquad (24)$$

Or

$$\mathbf{G_M = 19.2I_B} \qquad (25)$$





The two above plots show how GM of an OTA will vary with input voltage. The second plot is just more magnified around 0 volts which is where they are generally operated. What you will not is that even when the input range is limited to +/- 10mV, GM has already changed by 3% to 4% (closer to 3%). To keep the linearity to within 1%, you need to limit the input to +/- 6mV.