CHEEZY

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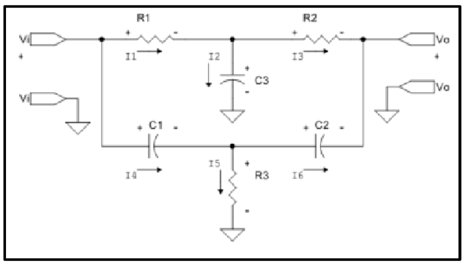
The Cheezy Synth Module Board uses the Twin T network quite extensively. So, in order to understand the Cheezy Synth we need to understand the Twin T. This is not as easy as it sounds. As simple as the twin T looks, it is a very complicated network. So one of the things we are going to look at in depth is just how the Twin T really works.

Twin T Networks



The twin T network is a rather unique RC network in that it can generated complex conjugates. Or in other words, you can get transfer functions that normally require inductors, without the use of inductors. This is why this network is used to make some rather special circuits. But first, we need to find out how this thing works. And despite the fact that this is a very simple little circuit, we are going to find that it is not quite so simple to analyze. So what we are going to do is start out with a special case where R1 = R2 = 2R3 and C1 = C2 = C3/2. Using these ratios will make a lot of things cancel out and we will bet a much simpler exp0resion for the transfer function. However, this transfer function will not be useful for our purposes, but this should make it much clearer what is going on.

We are going to use Kirkhoffs Voltage Law to write the equations for this circuit. This law states that the sum of the voltages around a loop must equal zero. Looking at the circuit we can see that there are four loops. Loop 1 consists of Vi, R1, C3. Loop 2 consists of Vo, R2, C3. Loop 3 consists of Vi, C1, R3. And finally, Loop 4 consists of Vo, C2, R3. You should note that many of the components are repeated in some of the loops.

The next thing you should not is the currents that are flowing through each component and the direction it is flowing. This is only convention. It does not matter which way you say the current is flowing as long as you are consistent. By defining which way the current is flowing, we can then decide on the polarity of the voltage across each component. this will help us to write the equations.

The values of the components are as follows:

C1 = C	R1 = R
C2 = C	R2 = R
C3 = 2C	R3 = R/2

We will now write the four loop equations.

$$V_i = i_1 R + \frac{i_2}{s2C}$$

$$V_o = \frac{i_2}{s2C} - i_3 R$$

$$V_i = \frac{i_4}{sC} + i_5 \frac{R}{2}$$

$$V_o = i_5 \frac{R}{2} - \frac{i_6}{sC}$$

What we have now are the basic equations. And we can begin to solve them, but first we can make some more observations about the circuit. Using Kirkhoffs Current law, which states that all of the currents flowing into and out of a node must sum to zero, we can write the following relationsips.

$$i_1 = i_2 + i_3$$

$$i_4 = i_5 + i_6$$

$$i_6 = -i_3$$

So, using the equations for the current, we can rewrite the equations for four loops.

$$V_i = (i_2 + i_3)R + \frac{i_2}{s2C}$$

$$V_o = \frac{i_2}{s2C} - i_3 R$$

$$V_i = \frac{i_5 + i_6}{sC} + i_5 \frac{R}{2}$$

$$V_o = i_5 \frac{R}{2} - \frac{i_6}{sC}$$

That was the first round of substitutions. We can also get rid of i_6 by substituting it with $-i_3$.

$$V_i = (i_2 + i_3)R + \frac{i_2}{s2C}$$

$$V_o = \frac{i_2}{s2C} - i_3 R$$

$$V_i = \frac{i_5 - i_3}{sC} + i_5 \frac{R}{2}$$

$$V_o = i_5 \frac{R}{2} + \frac{i_3}{sC}$$

So we have two fewer variables in our equations. It would be nice to get rid of one more if we could. I saw somebody use this trick. Vi + Vo = Vi +Vo, obviously. So we can do this.

$$(i_2 + i_3)R + \frac{i_2}{s2C} + \frac{i_2}{s2C} - i_3R = \frac{i_5 - i_3}{sC} + i_5\frac{R}{2} + i_5\frac{R}{2} + \frac{i_3}{sC}$$

$$i_2R + \frac{i_2}{sC} = \frac{i_5}{sC} + i_5R$$

Now multiply both sides by sC.

$$i_2sCR + i_2 = i_5 + i_5sCR$$

$$i_2(sCR + 1) = i_5(1 + sCR)$$

And we end up with this result.

$$i_2 = i_5$$

We can now use that result in our four equations.

$$V_i = (i_2 + i_3)R + \frac{i_2}{s2C}$$

$$V_o = \frac{i_2}{s2C} - i_3R$$

$$V_i = \frac{i_2 - i_3}{sC} + i_2\frac{R}{2}$$

$$V_o = i_2\frac{R}{2} + \frac{i_3}{sC}$$

This now greatly simplifies maters. First thing we note is that $V_O = V_O$ and $V_i = V_i$. Yeah, this may look obvious, but we can use this little observation to write the following two equations.

$$(i_2 + i_3)R + \frac{i_2}{s2C} = \frac{i_2 - i_3}{sC} + i_2\frac{R}{2}$$

$$i_2R + i_3R + \frac{i_2}{s2C} = \frac{i_2}{sC} - \frac{i_3}{sC} + i_2\frac{R}{2}$$

$$i_3R + \frac{i_3}{sC} = \frac{i_2}{sC} - \frac{i_2}{s2C} - i_2\frac{R}{2}$$

$$i_3\left(R + \frac{1}{sC}\right) = i_2\left(\frac{1}{sC} - \frac{1}{s2C} - \frac{R}{2}\right)$$

$$i_3\left(R + \frac{1}{sC}\right) = i_2\left(\frac{1}{s2C} - \frac{R}{2}\right)$$

$$i_3\left(\frac{sCR+1}{sC}\right)=i_2\left(\frac{1-sCR}{2sC}\right)$$

$$i_3=i_2\frac{\left(\frac{1-sCR}{2sC}\right)}{\left(\frac{sCR+1}{sC}\right)}$$

$$i_3=i_2\frac{1-sCR}{2sCR+2}$$

Now that we have an expression for i_3, we can substitute that back into two of our four equations. Let's use these two.

$$V_o=\frac{i_2}{s2C}-i_3R$$

$$V_i=(i_2+i_3)R+\frac{i_2}{s2C}$$

$$V_o=\frac{i_2}{s2C}-i_2\frac{1-sCR}{2sCR+2}R$$

$$V_i=\left(i_2+i_2\frac{1-sCR}{2sCR+2}\right)R+\frac{i_2}{s2C}$$

All we need to do now is to simplify these two equations.

$$V_o=i_2\left(\frac{1}{s2C}-\frac{1-sCR}{2sCR+2}R\right)$$

$$V_o=i_2\left(\frac{1}{s2C}-\frac{R-sCR^2}{2sCR+2}\right)$$

$$V_o=i_2\left(\frac{s2CR+2-s2CR+s^22C^2R^2}{4s^2C^2R+s4C}\right)$$

$$V_o=i_2\left(\frac{2+s^22C^2R^2}{4s^2C^2R+s4C}\right)$$

$$V_o=i_2\left(\frac{1+s^2C^2R^2}{2s^2C^2R+s2C}\right)$$

Now we do the same for V_i.

$$V_i=\left(i_2+i_2\frac{1-sCR}{2sCR+2}\right)R+\frac{i_2}{s2C}$$

$$V_i=i_2\left(\left(1+\frac{1-sCR}{2sCR+2}\right)R+\frac{1}{s2C}\right)$$

$$V_i = i_2\left(R + \frac{R - sCR^2}{2sCR + 2} + \frac{1}{s2C}\right)$$

This one is a little more complicated. The common denominator for the above is $4s^2C^2R+s4C$.

$$V_i = i_2\left(\frac{4s^2C^2R^2 + s4CR + s2CR - s^2 2C^2R^2 + 2sCR + 2}{4s^2C^2R + s4C}\right)$$

$$V_i = i_2\left(\frac{2s^2C^2R^2 + s8CR + 2}{4s^2C^2R + s4C}\right)$$

$$V_i = i_2\left(\frac{s^2C^2R^2 + s4CR + 1}{s^2 2C^2R + s2C}\right)$$

What we are looking for is the ration ov VO/Vi. So all we need to do is divide those two equations.

$$\frac{V_O}{V_i} = \frac{i_2\left(\frac{1 + s^2C^2R^2}{2s^2C^2R + s2C}\right)}{i_2\left(\frac{s^2C^2R^2 + s4CR + 1}{s^2 2C^2R + s2C}\right)}$$

And this readily simplifies down to:

$$\frac{V_O}{V_i} = \frac{1 + s^2C^2R^2}{s^2C^2R^2 + s4CR + 1}$$

We are almost there, but not quite. We need to multiply both the top and bottom of this ratio by $1/C^2R^2$ and this will get the equation into a form that we are a bit more familiar with.

$$\frac{V_O}{V_i} = \frac{s^2 + \frac{1}{C^2R^2}}{s^2 + s\frac{4}{CR} + \frac{1}{C^2R^2}}$$

This of course, is the solution to the simple case.